A FORTIORI REASONING IN JUDAIC LOGIC

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This paper consists of excerpts from the author's book Judaic Logic *(Geneva, 1995), with a few slight modifications. The full original text may be found at www.TheLogician.net.*

1. Historical Background

Logic in Judaism is mainly used for the determination and application of Jewish law, though also for the interpretation of the stories in holy texts. The founding document and proof-text of the Jewish faith and religion is, as is well known, the *Torah* (translated as the Law, or Doctrine). This refers to the Five Books of Moses or Pentateuch (*Chumash*, in Hebrew), which Judaism dates as 3,300 years old.

The Jewish Bible, or *Tanakh*, consists of this 5-volume Torah, together with the 8 other prophetic books and 11 other holy scriptures, written over the next 800 years or so. TaNaKh is an acronym, including the initials T of Torah, N of *Neviim* (Prophets) and K of *Ketuvim* (Scriptures); the books of the Bible other than those written by Moses are therefore simply known as the *Nakh*. The latter play a relatively secondary role in the development of Jewish law, being referred to occasionally to resolve certain questions of detail or to provide illustrations.

The *Talmud* (which means, teaching) is an enormous compilation of legal discussions between Rabbis, stretching over several centuries, starting about 2,100 years ago (at least). It includes two main components: the *Mishnah* (meaning, learning by repetition - pl. *Mishnaiot*), which was edited by R. Yehudah HaNassi in the 1st century CE, followed by the *Gemara* (meaning, completion - pl. *Gemarot*), which was redacted by R. Ashi in the 5th century. Actually, there are two Talmuds: the *Bavli* (or Babylonian), which is the one we just mentioned, and the parallel *Yerushalmi* (or Jerusalem), which was closed in Israel some 130 years earlier, in the 4th century, and carries relatively less authority. Nowadays, most editions of the Talmud include a mass of later commentaries and supercommentaries.

Jewish law, or the *Halakhah* (meaning, the Path, or the 'Way to go'), as it stands today, is the outcome of a long historical process of debate and practice, in which the above mentioned documents, mainly the Torah and the Talmud, have played the leading roles. Jewish law, note, concerns not only interactions between individuals (be they civil, commercial or criminal) and societal issues (communal or national structures and processes), but also the personal behavior of individuals (privately or in relation to God) and collective religious obligations (which may be carried out by selected individuals, such as the priests or Levites).

Talmudic law was decided, with reference to the Torah, after much debate. In a first stage, the debate crystallized as the Mishnah; in a later stage, as the Gemara. The methods used in such discourse to interpret the Torah document are known as 'hermeneutic' principles (or,

insofar as they are prescribed, rules). In Hebrew, they are called *midot* (sing. *midah*), meaning, literally, 'measures' or 'virtues'. This Talmudic 'logic' has certain specificities, both in comparison to generic logic and intramurally in the way of distinct tendencies in diverse schools of thought. Various Rabbis proposed diverse collections of such methodological guidelines, intending thereby to explain and justify legal decision-making.

The earliest compilations were: the Seven Rules of Hillel haZaken (1st century BCE); the Thirteen Rules of Rabbi Ishmael ben Elisha (2nd century CE); and the Thirty-two Rules of Rabbi Eliezer ben Yose haGelili, of slightly later date. These lists are given as Baraitot, the first two in the introductory chapter to the *Sifra* (1:7), a Halakhic commentary to *Leviticus*, also known as *Torat Kohanim*, attributed to R. Yehudah b. Ilayi, a disciple of R. Akiba (2nd cent. CE), and the third within later works. Baraitot were legal rulings by Tanaim not included in the Mishnah; but they were regarded in the Gemara as of almost equal authority[1].

Judaic logic has long used and explicitly recognized a form of argument called *qal vachomer* (meaning, lenient and stringent). This is the first and most deductive of the hermeneutic principles listed by the Rabbis. According to *Genesis Rabbah* (92:7), an authoritative Midrashic work, there are ten samples of such of argument in the Tanakh: of which four occur in the Torah, and another six in the Nakh. Countless more exercises of *qal vachomer* reasoning appear in the Talmud, usually signaled by use of the expression *kol sheken*. Hillel and Rabbi Ishmael ben Elisha include this heading in their respective lists of hermeneutic principles, and much has been written about it since then.

In English discourse, such arguments are called *a-fortiori* (*ratione*, Latin; meaning, with stronger reason) and are usually signaled by use of the expression *all the more*. The existence of a Latin, and then English, terminology suggests that Christian scholars, too, eventually found such argument worthy of study (influenced no doubt by the Rabbinical precedent)[2]. But what is rather interesting, is that modern secular treatises on formal logic all but completely ignore it - which suggests that no decisive progress was ever achieved in analyzing its precise morphology. Their understanding of a-fortiori argument is still today very sketchy; they are far from the formal clarity of syllogistic theory.

What seems obvious at the outset, is that a-fortiori logic is in some way concerned with the *quantitative* and not merely the qualitative description of phenomena. Aristotelian syllogism deals with attributes of various kinds, without effective reference to their *measures or degrees*; it serves to classify attributes in a hierarchy of species and genera, but it does not place these attributes in any intrinsically numerical relationships. The only "quantity" which concerns it, is the extrinsic count of the instances to which a given relationship applies (which makes a proposition general, singular or particular).

This is very interesting, because - as is well known to students of the history of science - modern science arose precisely through the growing awareness of quantitative issues. Before the Renaissance, measurement played a relatively minimal role in the physical sciences; things were observed (if at all) mainly with regard to their qualitative similarities and differences. Things were, say, classed as hot or cold, light or heavy, without much

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1   As Scherman has pointed out, these Baraitot were different, in that they were not in themselves statements of law but explanations of how the laws were derived from the Torah source.

2   There are already, in the Christian Bible, examples of a-fortiori, some of which are analyzed by H. Maccoby in *The Mythmaker: Paul and the Invention of Christianity*. The author mentions Paul's fondness for the argument, but shows him to have lacked knowledge of the *'dayo* principle' (see further on), concluding that his use of the form was more akin to the rhetoric of Hellenistic Stoic preachers (pp. 64-67).

further precision. Modern science introduced physical instruments and mathematical tools, which enabled a more fine-tuned pursuit of knowledge in the physical realm.

A-fortiori argument may well constitute the formal bridge between these two methodological approaches. Its existence in antiquity, certainly in Biblical and Talmudic times, shows that quantitative analysis was not entirely absent from the thought processes of the precursors of modern science. They may have been relatively inaccurate in their measurements, their linguistic and logical equipment may have been inferior to that provided by mathematical equations, but they surely had some knowledge of quantitative issues.

In the way of a side note, I would like to here make some comments about the history of logic. Historians of logic must in general distinguish between several aspects of the issue.

(a) The *art or practice* of logic: as an act of the human mind, an insight into the relations between things or ideas, logic is part of the natural heritage of all human beings; it would be impossible for us to perform most of our daily tasks or to make decisions without some exercise of this conceptual power. I tend to believe that all forms of reasoning are natural; but it is not inconceivable that anthropologists demonstrate that such and such a form was more commonly practiced in one culture than any other[3], or first appeared in a certain time and place, or was totally absent in a certain civilization.

(b) The *theoretical awareness and teaching* of logic: at what point in history did human beings become self-conscious in their use of reasoning, and began to at least orally pass on their thoughts on the subject, is a moot question. Logic can be grasped and discussed in many ways; and not only by the formal-symbolic method, and not only in writing. Also, the question can be posed not only generally, but with regard to specific forms of argument. The question is by definition hard for historians to answer, to the extent that they can only rely on documentary evidence in forming judgments. But orally transmitted traditions or ancient legends may provide acceptable clues.

(c) The *written science* of Logic, as we know it: the documentary evidence (his written works, which are still almost totally extant) points to Aristotle (4th century BCE) as the first man who thought to use symbols in place of terms, for the purpose of analyzing various eductive and syllogistic arguments, involving the main forms of categorical proposition. Since then, the scope of formal logic has of course greatly broadened, thanks in large measure to Aristotle's admirable example, and findings have been systematized in manifold ways.

Some historians of logic seem to equate the subject exclusively with its third, most formal and literary, aspect (see, for instance, Windelband, or the *Encyclopaedia Britannica* article on the subject). But, even with reference only to Greek logic, this is a very limiting approach. Much use and discussion of logic preceded the Aristotelian breakthrough, according to the reports of later writers (including Aristotle). Thus, the Zeno paradoxes were a clear-minded use of Paradoxical logic (though not a theory concerning it). Or again, Socrates' discussions (reported by his student Plato) about the process of Definition may be classed as logic theorizing, though not of a formal kind.

Note that granting a-fortiori argument to be a natural movement of thought for human beings, and not a peculiarly Jewish phenomenon, it would not surprise me if documentary evidence of its use were found in Greek literature (which dates from the 5th century BCE)

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3 I have an impression, for instance, that modern French discourse involves more use of a-fortiori than modern English discourse. To what extent that is true, and why it should be so, I cannot venture to say.

or its reported oral antecedents (since the 8th century); but, so far as I know, Greek logicians - including Aristotle - never developed a formal and systematic study of it.

One of the dogmas of the Jewish faith is that the hermeneutic principles it uses (including the a-fortiori argument) were part of the oral traditions handed down to Moses at Sinai, together with the written Torah. What is obvious is that the Torah is a complex document which could never be understood without the mental exercise of some logical intuitions. A people who over a thousand years before the Greeks had a written language, could well also have early on used a set of logical techniques such as the hermeneutic principles. These were not, admittedly, logic theories as formal as Aristotle's; but they were still effective. They do not, it is true, appear to have been put in writing until Talmudic times; but that does not definitely prove that they were not in use and orally discussed long before.

With regard to the suggestion by some historians that the rabbinic interest in logic was a result of a Greek cultural influence - one could equally argue the reverse, that the Greeks were awakened to the issues of logic by the Jews. The interactions of people always involve some give and take of information and methods; the question is only who gave what to whom and who got what from whom. The mere existence of a contact does not in itself answer that specific question; it can only be answered with reference to a wider context.

A case in point, which serves to illustrate and prove our contention of the independence of Judaic logic, is precisely the *qal vachomer* argument. The Torah provides documentary evidence that this form of argument was at least *used* at the time it was written, indeed two centuries earlier (when the story of Joseph and his brothers, which it reports, took place). If we rely only on documentary evidence, the written report in Talmudic literature, the conscious and explicit *discussion* of such form of argument must be dated to at least the time of Hillel, and be regarded as a ground-breaking discovery.

To my knowledge, my *Judaic Logic* study is the first ever thorough analysis of *qal vachomer* argument, using the Aristotelian method of symbolization of terms (or theses). The identification of the varieties of the argument, and of the significant differences between subjectal (or antecedental) and predicatal (or consequental) forms of it, seems also to be novel.

2.   The Valid Moods of A Fortiori

A-fortiori logic was admittedly a hard nut to crack; it took me two or three weeks to break the code. The way I did it, back in the early 1990's, was to painstakingly analyze a dozen concrete Biblical and Talmudic examples, trying out a great many symbolic representations, until I discerned all the factors involved in them. It was not clear, at first, whether all the arguments are structurally identical, or whether there are different varieties. When a few of the forms became transparent, the rest followed by the demands of symmetry. Validation procedures, formal limitations and derivative arguments could then be analyzed with relatively little difficulty.

Although this work was largely independent and original, I am bound to recognize that it was preceded by considerable contributions by past Jewish logicians, and in celebration of this fact, illustrations given here will mainly be drawn from Judaic sources.

Let us begin by listing and naming all the *valid* moods of a-fortiori argument in abstract form; we shall have occasion later to consider examples. We shall adopt a terminology

which is as close to traditional as possible, but it must be kept in mind that the old names used here may have new senses (in comparison to, say, their senses in syllogistic theory), and that some neologisms are inevitable in view of the novelty of our discoveries.

The formalities of a-fortiori logic are important, not only to people interested in Talmudic logic, but to logicians in general; for the function of the discipline of logic is to identify, study, and validate, all forms of human thought. And it should be evident with little reflection that we commonly use reasoning of this kind in our thinking and conversation; and indeed its essential message is well known and very important to modern science.

An explicit a-fortiori argument always involves three propositions, and *four* terms. We shall call the propositions: the major premise, the minor premise, and the conclusion, and always list them in that order. The terms shall be referred to as: the major term (symbol, **P**, say), the minor term (**Q**, say), the middle term (**R**, say), and the subsidiary term (**S**, say). In practice, the major premise is very often left unstated; and likewise, the middle term.

A-fortiori argument can be represented by a *triangular star*, at the center of which is the middle item (R) through which the three other items, P, Q, and S are related to each other.
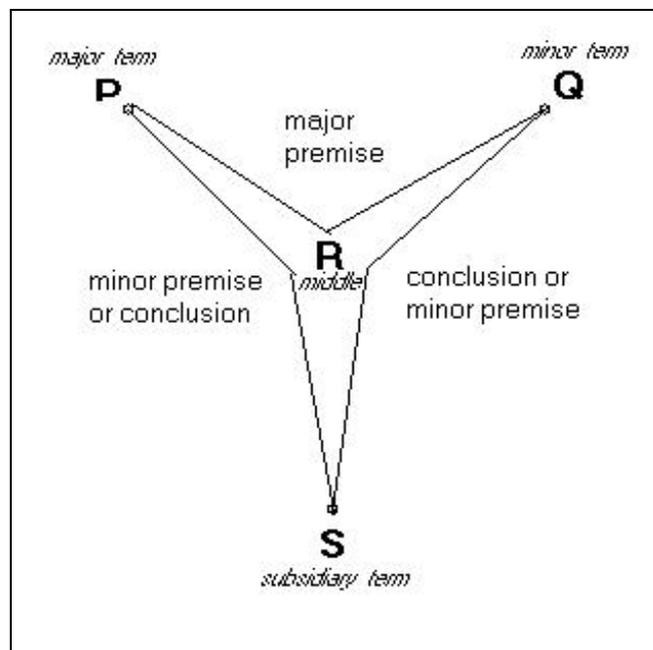
There are, it turns out, many varieties of a-fortiori argument. The following table classifies its primary forms (the secondary forms are derived from these but not included in it, or mentioned anymore in this paper):

FORM	STRUCTURE	
Copulative	(1) Subjectal	(2) Predicatal
Implicational	(3) Antecedental	(4) Consequental
POLARITY	ORIENTATION	
(a) Positive	Minor to major	Major to minor
(b) Negative	Major to minor	Minor to major

We shall here only analyze "copulative" forms of the argument. There are essentially four valid moods. Two of them are subjectal in structure and two of them predicatal in structure; and for each structure, one of the arguments is positive in polarity and the other is negative. "Implicational" forms of the a-fortiori argument are *essentially similar in structure* to the above copulative forms, except that they are more broadly designed to concern theses (propositions), rather than terms. The relationship involved is consequently one of implication, rather than one of predication; that is, we find in them the expression "implies", rather than the copula "is". Fuller treatment of implicational forms may of course be found in my book *Judaic Logic*.

SUBJECTAL moods.

(1) Positive version. (Minor to major.)

> P is more R than Q (is R),
>
> and, Q is R enough to be S;
>
> therefore, all the more, P is R enough to be S.

A similar argument with P in the minor premise and Q in the conclusion ("major to minor") would be invalid.

(2) Negative version. (Major to minor.)

> P is more R than Q (is R),
>
> yet, P is *not* R enough to be S;
>
> therefore, all the more, Q is *not* R enough to be S.

A similar argument with Q in the minor premise and P in the conclusion ("minor to major") would be invalid.

PREDICATAL moods.

(3) Positive version. (Major to minor.)

> More R is required to be P than to be Q,
>
> and, S is R enough to be P;
>
> therefore, all the more, S is R enough to be Q.

A similar argument with Q in the minor premise and P in the conclusion ("minor to major") would be invalid.

(4) Negative version. (Minor to major.)

> More R is required to be P than to be Q,
>
> yet, S is *not* R enough to be Q;
>
> therefore, all the more, S is *not* R enough to be P.

A similar argument with P in the minor premise and Q in the conclusion ("major to minor") would be invalid.

Once examined in their symbolic purity, the arguments listed above all appear as *intuitively obvious*: they 'make sense'. We can, additionally, easily convince ourselves of their logical correctness, through a visual image as in Cartesian geometry. Represent R by a line, and place points P and Q along it, P being further along the line than Q - all the arguments follow by simple mathematics. The pictorial representation is in fact made with reference to the comparative propositions that underlie such arguments, ordering items P, Q, and S, according to their position in a *common continuum* R, as follows:

The a fortiori argument orders items P, Q, S, according to their position in a common continuum R
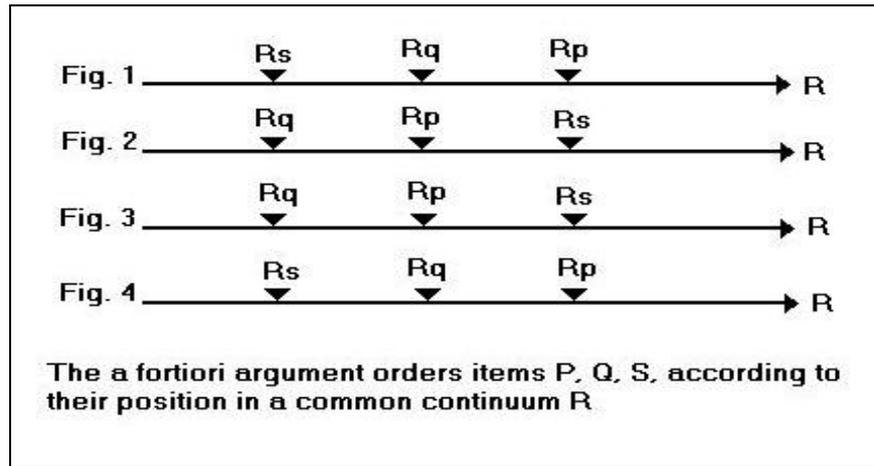
The above information can be summarized as follows:

Figure 1	Figure 2	Figure 3	Figure 4
Rp > Rq	Rp > Rq	Rp > Rq	Rp > Rq
Rq > Rs	Rp < Rs	Rs > Rp	Rs < Rq
So, Rp > Rs	So, Rq < Rs	So, Rs > Rq	So, Rs < Rp

Here, of course, ">" means "is more than" and "<" means "is less than". (Note that the four figures of a-fortiori should not be confused with the four of syllogism they imply, which are, in the order shown: the fourth, third, first and second figures.)

The expression "all the more" used with the conclusion is intended to connote that the inferred proposition is more "forceful" than the minor premise, as well as suggest the quantitative basis of the inference (i.e. that it is a-fortiori). Note that instead of the words "and" or "yet" used to introduce the minor premise, we could just as well have used the expression "nonetheless", which seems to balance nicely with the phrase "all the more".

The role of the major premise is always to relate the major and minor terms (P and Q) to the middle term (R); the middle term serves to place the major and minor terms along a quantitative continuum. The major premise is, then, a kind of *comparative* proposition of some breadth, which will make possible the inference concerned; note well that it contains three of the terms, and that its polarity is always positive (this will be demonstrated further down). The term which signifies a greater measure or degree (more) within that range, is immediately labeled the major; the term which signifies a smaller measure or degree (less) within that range, is immediately labeled the minor (these are conventions, of course). P and Q may also conveniently be called the "extremes" (without, however, intending that they signify extreme quantities of R).

Note that here, unlike in syllogism, the major premise involves both of the extreme terms and the minor premise may concern either of them; thus, the expressions major and minor terms, here, have a different value than in syllogism, it being the relative content of the terms which determines the appellation, rather than position within the argument as a whole. Furthermore, the middle term appears in all three propositions, not just the two premises.

The function of the minor premise is to positively or negatively relate one of the extreme terms to the middle and subsidiary terms; the conclusion thereby infers *a similar relation*

for the remaining extreme. If the minor premise is positive, so is the conclusion; such moods are labeled positive, or *modus ponens* in Latin; if the minor premise is negative, so is the conclusion; such moods are labeled negative, or *modus tollens*. Note well that the minor premise may concern either the major or the minor term, as the case may be. Thus, the inference may be "from major (term, in the minor premise) to minor (term, in the conclusion)" - this is known as inference *a majori ad minus*; or in the reverse case, "from minor (term, in the minor premise) to major (term, in the conclusion)" - this is called *a minori ad majus*.

There are *notable differences between subjectal and predicatal a-fortiori*. In subjectal argument, the extreme terms have the logical role of subjects, in all three propositions; whereas, in predicatal argument, they have the role of predicates. Accordingly, the subsidiary term is the predicate of the minor premise and conclusion in subjectal a-fortiori, and their subject in predicatal a-fortiori.

Because of the functional difference of the extremes, the arguments have opposite orientations. In subjectal argument, the positive mood goes from minor to major, and the negative mood goes from major to minor. In predicatal argument, the positive mood goes from major to minor, and the negative mood goes from minor to major. The symmetry of the whole theory suggests that it is exhaustive.

With regard to the above mentioned *invalid* moods, namely major-to-minor positive subjectals or negative predicatals, and minor-to-major negative subjectals or positive predicatals, it should be noted that the premises and conclusion are not in conflict. The invalidity involved is that of a non-sequitur, and not that of an antinomy. It follows that such arguments, though deductively valueless, can, eventually, play a small inductive role (just as invalid apodoses are used in adduction).

A couple of comments, which concern all forms of the argument, still need to be made.

The standard form of the major premise is a comparative proposition with the expression "*more... than*" (superior form). But we could just as well *commute* such major premises, and put them in the "*less... than*" form (inferior form), provided we accordingly reverse the order in it of the terms P and Q. Thus, 'P is more R than Q' could be written 'Q is less R than P', 'More R is required to be P than to be Q' as 'Less R is required to be Q than to be P', and similarly for implicational forms, without affecting the arguments. These are mere eductions (the propositions concerned are equivalent, they imply each other and likewise their contradictories imply each other), without fundamental significance; but it is well to acknowledge them, as they often happen in practice and one could be misled. The important thing is always is to know which of the terms is the major (more R) and which is the minor (less R).

Also, it should also be obvious that the major premise could equally have been an egalitarian one, of the form "*as much... as*" (e.g. 'P is as much R as Q (is R)'). The arguments would work equally well (P and Q being equivalent in them). However, in such cases it would not be appropriate to say "all the more" with the conclusion; but rather use the phrase "*just as much*". Nevertheless, we must regard such arguments as still, in the limit, a-fortiori in structure. The expression "all the more" is strictly-speaking a redundancy, and serves only to signal that a specifically a-fortiori kind of inference is involved; we could equally well everywhere use the word "therefore", which signifies for us that an inference is taking place, though it does not specify what kind.

It follows that each of the moods listed above stands for three valid moods: the superior (listed), and corresponding inferior and egalitarian moods (unlisted).

Lastly, it is important to keep in mind, though obvious, that the form 'P is more R than Q' means 'P is more R than Q *is R*' (in which Q is as much a subject as P, and R is a common

predicate), and should not be interpreted as 'P is more R than *P is* Q' (in which P is the only subject, common to two predicates Q and R, which are commensurable in some unstated way, such as in spatial or temporal frequency, allowing comparison between the degrees to which they apply to P). In the latter case, R cannot serve as middle term, and the argument would not constitute an a-fortiori. Formal ambiguities of this sort can lead to fallacious a-fortiori reasoning.

Detailed formal validation of valid moods, and invalidation of invalid ones, are of course essential, but will not be undertaken in this short paper. Suffices to say that the propositions colloquially used as premises and conclusions of a-fortiori arguments are entirely reducible to known forms, namely (where X, Y are any terms or theses, as the case may be) to categoricals ('X is Y', 'X is not Y'), conditionals ('if X then Y', ' if X not-then Y') and comparatives (X > or = or < Y, or their negations; and X ⊃ Y, or its negation). Consequently, a-fortiori arguments may be systematically explicated and validated by such reductions.

Many additional details and issues, some of them quite important, are omitted here for the sake of brevity.

3.   Samples in the Torah

Our first job was to *formalize* a-fortiori arguments, to try and express them in symbolic terms, so as to abstract from their specific contents what it is that makes them seem "logical" to us. We needed to show that there are legitimate forms of such argument, which are not mere flourishes of rhetoric designed to cunningly mislead, but whose function is to guide the person(s) they are addressed to through genuinely inferential thought processes. This we have done in the previous section.

Let us now, with reference to cogent examples, check and see how widely applicable our theory of the *qal vachomer* argument is thus far, or whether perhaps there are new lessons to be learnt. I will try and make the reasoning involved as transparent as possible, step by step. The reader will see here the beauty and utility of the symbolic method inaugurated by Aristotle.

Biblical a-fortiori arguments generally seem to consist of a minor premise and conclusion; they are presented without a major premise. They are worded in typically Jewish fashion, as a question: "this and that, how much more so and so?" The question mark (which is of course absent in written Biblical Hebrew, though presumably expressed in the tone of speech) here serves to signal that no other conclusion than the one suggested could be drawn; the rhetorical question is really "do you think that another conclusion could be drawn? no!"

Concerning the absence of a major premise, it is well known and accepted in logic theorizing that arguments are in practice not always fully explicit (*meforash*, in Hebrew); either one of the premises and/or the conclusion may be left tacit (*satum*, in Hebrew). This was known to Aristotle, and did not prevent him from developing his theory of the syllogism. We naturally tend to suppress parts of our discourse to avoid stating "the obvious" or making tiresome repetitions; we consider that the context makes clear what we intend. Such incomplete arguments, by the way, are known as *enthymemes* (the word is of Greek origin).

The missing major premise is, in effect, *latent* in the given minor premise and conclusion; for, granting that they are intended in the way of an argument, rather than merely a statement of fact combined with an independent question, it is easy for any reasonably intelligent person to *construct* the missing major premise, if only subconsciously. If the middle term is already explicit in the original text, this process is relatively simple. In some cases, however, no middle term is immediately apparent, and we must provide one (however intangible) which verifies the argument.

In such case, we examine the given major and minor terms, and abstract from them a concept, which seems to be their common factor. To constitute an appropriate middle term, this underlying concept must be such that it provides a quantitative continuum along which the major and minor terms may be placed. Effectively, we syllogistically substitute two degrees of the postulated middle term, for the received extreme terms. Note that a similar operation is sometimes required, to standardize a subsidiary term which is somewhat disparate in the original minor premise and conclusion.

We are logically free to volunteer any credible middle term; in practice, we often do not even bother to explicitly do so, but just take for granted that one exists. Of course, this does not mean that the matter is entirely arbitrary. In some cases, there may in fact be no appropriate middle term; in which case, the argument is simply fallacious (since it lacks a major premise). But normally, no valid middle term is explicitly provided, on the understanding that one is easy to find - there may indeed be many obvious alternatives to choose from (and this is what gives the selection process a certain liberty).

(1) Let us begin our analysis with a Biblical sample of the simplest form of *qal vachomer*, subjectal in structure and of positive polarity. It is the third occurrence of the argument in the *Chumash*, or Pentateuch (Numbers, 12:14). God has just struck Miriam with a sort of leprosy for speaking against her brother, Moses; the latter beseeches God to heal her; and God answers:

> *If her father had but spit in her face, should she not hide in shame seven days? let her be shut up without the camp seven days, and after that she shall be brought in again.*

If we reword the argument in standard form, and make explicit what seems to be tacit, we obtain the following.

Major premise:

> "Divine disapproval (here expressed by the punishment of leprosy)" (=P) is more "serious disapproval" (=R) than "paternal disapproval (signified by a spit in the face)" (=Q);

Minor premise:

> if paternal disapproval (Q) is serious (R) enough to "cause one to be in isolation (hide) in shame for seven days" (=S),

Conclusion:

> then Divine disapproval (P) is serious (R) enough to "cause one to be in isolation (be shut up) in shame for seven days" (=S).

Note that the middle term (seriousness of disapproval) was not explicit, but was conceived as the common feature of the given minor term (father's spitting in the face) and major term (Gd afflicting with leprosy). Concerning the subsidiary term these propositions have in common, note that it is not exactly identical in the two original sentences; we made it uniform by replacing the differentia (hiding and being shut up) with their commonalty

(being in isolation). More will be said about the specification "for seven days" in the subsidiary term (S), later.

(2) A good Biblical sample of negative subjectal *qal vachomer* is that in Exodus, 6:12 (it is the second in the Pentateuch). God tells Moses to go back to Pharaoh, and demand the release of the children of Israel; Moses replies:

> *Behold, the children of Israel have not hearkened unto me; how then shall Pharaoh hear me, who am of uncircumcised lips?*

This argument may be may be construed to have run as follows:

Major premise:

> The children of Israel (=P) "fear God" (=R) more than Pharaoh (=Q) does;

Minor premise:

> yet, they (P) did not fear God (R) enough to hearken unto Moses (=S);

Conclusion:

> all the more, Pharaoh (Q) will not fear God (R) enough to hear Moses (S).

Here again, we were only originally provided with a minor premise and conclusion; but their structural significance (two subjects, a common predicate) and polarity were immediately clear. The major premise, however, had to be constructed; we used a middle term which seemed appropriate - "fear of God".

Concerning our choice of middle term. The interjection by Moses, "I am of uncircumcised lips", which refers to his speech problem (he stuttered), does not seem to be the intermediary we needed, for the simple reason that this quality does not differ in degree in the two cases at hand (unless we consider that Moses expected to stutter more with Pharaoh than he did with the children of Israel). Moses' reference to a speech problem seems to be incidental - a rather lame excuse, motivated by his characteristic humility - since we know that his brother Aaron acted as his mouthpiece in such encounters.

In any case, note in passing that the implicit intent of Moses' argument was to dissuade God from sending him on a mission. Thus, an additional argument is involved here, namely: "since Pharaoh will not hear me, there is no utility in my going to him" - but this is not a *qal vachomer*.

(3) The first occurrence of *qal vachomer* in the Torah - and perhaps historically, in any extant written document - is to be found in Genesis, 44:8 (it thus dates from the Patriarchal period, note). It is a positive predicatal a-fortiori. Joseph's brothers are accused by his steward of stealing a silver goblet, and they retort:

> *Behold, the money, which we found in our sacks' mouths, we brought back unto thee out of the land of Canaan; how then should we steal out of thy lord's house silver or gold?*

According to our theory, the argument ran as follows:

Major premise:

> You will agree to the general principle that more "honesty" (=R) is required to return found money (=P) than to refrain from stealing a silver goblet (=Q);

Minor premise:

> and yet, we (=S) were honest (R) enough to return found money (P);

Conclusion:

therefore, you can be sure that we (S) were honest (R) enough to not-steal the silver goblet (Q).

Here again, the middle term (honesty) was only implicit in the original text. The major premise may be true because the amount of money involved was greater than the value of the silver goblet, or because the money was found (and might therefore be kept on the principle of "finders keepers") whereas the goblet was stolen; or because the positive act of returning something is superior to a mere restraint from stealing something.

(4) There is no example of negative predicatal a-fortiori in the Torah; but I will recast the argument in Deuteronomy, 31:27, so as to illustrate this form. The original argument is in fact positive predicatal in form, and it is the fourth and last example of *qal vachomer* in the Pentateuch:

> *For I know thy rebellion, and thy stiff neck; behold, while I am yet alive with you this day, ye have been rebellious against the Lrd; and how much more after my death?*

We may reword it as follows, for our purpose:

Major premise:

> More "self-discipline" (=R) is required to obey God in the absence of His emissary, Moses (=P), than in his presence (=Q);

Minor premise:

> the children of Israel (=S) were not sufficiently self-disciplined (R) to obey God during Moses' life (Q);

Conclusion:

> therefore, they (S) would surely lack the necessary self-discipline (R) after his death (P).

In this case, note, the middle term was effectively given in the text; "self-discipline" is merely the contrary of disobedience, which is implied by "stiff neck and rebelliousness". The constructed major premise is common sense.

We have thus illustrated all four moods of copulative *qal vachomer* argument, with the four cases found in the Torah. We can similarly analyze and classify the six cases which according to the Midrash occur in the other books of the Bible. In every case, the major premise is tacit, and must be made up. The cases are:

- Samuel I, 23:3. This is a positive antecedental.
- Jeremiah, 12:5. This is a positive antecedental (in fact, there are two arguments with the same thrust, here).
- Ezekiel, 15:5. This is a negative subjectal.
- Proverbs, 11:31. This is a positive subjectal.
- Esther, 9:12. This is a positive antecedental (if at all an a-fortiori, see discussion in a later chapter).

The following is a quick and easy way to classify any Biblical example of *qal vachomer*:

(a) What is the polarity of the given sentences? If they are positive, the argument is a *modus ponens*; if negative, the argument is a *modus tollens*.

(b) Which of the sentences contains the major term, and which the minor term? If the minor premise has the greater extreme and the conclusion has the lesser extreme, the argument is *a majori ad minus*; in the reverse case, it is *a minori ad majus*.

(c) Now, combine the answers to the two previous questions: if the argument is positive and minor to major, or negative and major to minor, it is subjectal or antecedental; if the argument is positive and major to minor, or negative and minor to major, it is predicatal or consequental.

(d) Lastly, decide by closer scrutiny, or trial and error, whether the argument is specifically copulative or implicational. At this stage, one is already constructing a major premise.

4.   The *Dayo* Principle

Rabbinical logicians raised an important question in relation to certain *qal vachomer* arguments. For instance, in the argument about Miriam (which we analyzed in the previous section), the minor premise posits a punishment of seven days for a relatively lesser crime, and the conclusion likewise posits a punishment of seven days for a relatively greater crime. Why only seven days? they wondered; should not the punishment be more, *proportionately* to the severity of the crime? A reasonable question.

Since the sample argument is of Divine origin, some Rabbis postulated that it suggests a universal logical rule, namely that the conclusion of a *qal vachomer* can never go further than the minor premise, in the specification of the measure or degree of the terms involved[4]. They called this, the *dayo* (sufficiency) principle (see *Baba Qama*, 2:5). Other Rabbis, like R. Tarphon (in *Baba Qama*, 25a), did not concur, but regarded a proportionate inference as permissible, at least in some cases. For my part, I would like to say the following.

In the argument concerning Miriam, it can easily be countered that God sentenced her in the conclusion to only seven days incarceration out of sheer mercy, though she might have been strictly-speaking subject to infinitely more; and that in any case, the seven days mentioned in the minor premise are not known through natural human insight, but equally through Divine fiat. Thus, this example does not by itself resolve the issue incontrovertibly.

Note, however, that the quantitative factor at issue may be made to stand somewhat outside the regular terms of the a-fortiori argument as such. It is not the quantitative difference between the major and minor terms which is at issue; that is already given (or taken for granted) in the major premise. What is at issue is a quantitative evaluation of the remaining terms, the middle term and the subsidiary term, as they appear in the minor premise and conclusion.

According to our theory (which is not all included in the present paper, remember), the outward uniformity of these terms in those propositions is a formal feature of a-fortiori argument. But this feature does not in itself exclude variety at a deeper level. Such specific differences are side-issues which the a-fortiori argument itself cannot prejudge. It takes supplementary propositions, in a separate argument, which is not a-fortiori but purely

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[4]   The principle is stated as *din leba min hadin lihiot benadon.* Note that the *Jewish Encyclopedia* translates this as "the conclusion of an argument is satisfied when it is like the major premise"; but what they mean by 'major premise' is what we here, more precisely, name 'minor premise'.

mathematical in form, to make inferences about the precise quantitative ramifications of the a-fortiori conclusion.

Thus, we may acknowledge the *dayo* principle as correct, provided it is understood as being a minimal position. It does not insist on the quantitative equality of the subsidiary or middle term (as the case may be) in the conclusion and minor premise, nor does it interdict an inequality; it merely leaves the matter open for further research. A-fortiori argument *per se* does not answer the question; it is from a formal point of view as compatible with equality as with inequality. To answer the question, additional information and other arguments must be sought. This is a reasonable solution.

Generally speaking, what is needed ideally is some mathematical formula which captures the *concomitant variation* between a term external to the a-fortiori argument as such (e.g. amount of punishment), and a term of variable value implicit in the a-fortiori (e.g. severity of the sin). This formula then stands as the major premise in a distinct argument, whose minor premise and conclusion contain the indefinite term at issue in the a-fortiori argument (the middle or subsidiary term, as the case may be, to repeat) as their common subject, and the said external term's values as their respective predicates.

There is no guarantee, note well, that the variation in the major premise will be an arithmetical proportionality; it could just as well be an inverse proportionality or a much more complex mathematical relationship, even one involving other variables. This is why the a-fortiori argument as such cannot predict the result; its premises lack the information required for a more refined conclusion. In some cases, the concomitance is simple and well known, and for this reason seems to be an integral part of the a-fortiori; but this is an illusion, the proof being that it does not always work, and in more complex cases a separate judgment must be made.

Let us now analyze the issue underlying the *dayo* principle in more formal terms. Consider a positive subjectal a-fortiori, whose subsidiary term (S) is a conjunction of two factors, a constant (say, K) and a variable (say, V); and suppose V is a function (f) of the middle term (R), i.e. that $V = f(R)$ in mathematical language. On a superficial level, the argument is simply as follows:

> P is more R than Q,
> and, Q is R enough to be S;
> therefore, P is R enough to be S.

But "R enough" is a threshold, it is not a fixed quantity. In the case of the minor premise, involving Q, the value of R is Rq, say; whereas, in the case of the conclusion, involving P, the value of R is Rp, say; and we know from the major premise that Rp is greater than Rq. Looking now at S, it is evident that if it consists only of a constant (K), it will be identical in the minor premise and the conclusion. But, if S involves a variable V, where V is a function of R, then S is not necessarily exactly the same in both propositions. If $V = f(R)$ represents a straightforward linear relationship, then Vp = f(Rp) will predictably be proportionately greater than Vq = f(Rq); but if $V = f(R)$ represents a more complicated relationship, then Vp = f(Rp) may be more or less than Vq = f(Rq), or equal to it, depending on the specifics of the formula.

Similar comments can be made with regard to the other valid moods of *qal vachomer*. Note in any case that all this is well and good in principle; but *in practice*, we may not be able to provide an appropriate and accurate mathematical equation. Some phenomena are

difficult and even impossible to measure; we may know that they somehow vary, but we may have no instruments with which to determine the variations, precisely or at all.

5.   Objections!

The formalization of a-fortiori argument has been found difficult by past logicians for various reasons. (a) The complexity and variety of the propositional forms involved. (b) There are many varieties of the argument. (c) Known samples are usually incompletely formulated. (d) Known samples often intertwine a mixture of purely a-fortiori and other forms of deductive inference. (e) The deductive and inductive issues were not adequately separated. We will clarify these matters in the present section.

Thus far, our goal has been to discover the essential form(s) of a-fortiori argument. We found the various kinds of premises and conclusion which ideally constitute such movements of thought. As in all formal logic, the conclusion *follows from* the premises; if the premises are true, then the conclusion is true. The presentation of a form of argument as valid does not in itself guarantee the truth of the premises. If any or all of the premises are not true, then the conclusion does not follow; the conclusion may happen to be false too, or it may be true for other reasons, but it is in any case a *non sequitur*.

This understanding of the relationship of premises and conclusion is not a special dispensation granted to our theory of a-fortiori, but applies equally well to all inference, be it eductive, syllogistic or otherwise deductive, or even inductive. In all cases, the question arises: *how are the premises themselves known?* And the answer is always: by any of the means legitimatized by the science of logic. A premise may be derived from experience by inductive arguments of various kinds, or be a logical axiom in the sense that their contradictories are self-denying, or even be Divinely revealed; or it may be deductively inferred in one way or another from such relatively primary propositions (whether they are *a posteriori* or *a priori*, to use the language of philosophers).

This issue has been acknowledged in the literature on Talmudic logic, through the doctrine of *objection* (in Hebrew, *teshuvah*; in Aramaic, *pirka*). A given a-fortiori argument, indeed any argument, may be criticized on formal grounds, if it is shown not to constitute a valid mood of reasoning. But it may also be objected to on material grounds, by demonstrating one or both of its premises is/are wholly or partly false, or at least open to serious doubt. The deduction as such may be valid, but its inductive backing (in the widest sense) may be open to doubt.

Consider for examples the Biblical samples of *qal vachomer* we have used as our illustrations.

In the argument concerning Miriam, we were given two sentences, neither of which is in itself obvious. Assuming that the Biblical verse as a whole is indeed intended as an argument, and not as two unrelated assertions, we may regard the first as a Divinely guaranteed truth and use it as our minor premise, but the second must somehow emerge as a conclusion. However, the major premise, which we ourselves construct to complete the argument, is in principle not indubitable. The one we postulated happens to seem reasonable (i.e. appears to be consistent with the rest of our knowledge); but it is conceivable that some objection could eventually be raised concerning it (say, that God attaches more importance to sins against parents than to sins against Himself).

In the next argument, by Moses, the major and minor premises are both known by empirical means. The former is a generalization, based on the past behavior patterns of the

children of Israel and Pharaoh; and the latter is a statement concerning more recent events. These propositions happen to be true, so that the conclusion is justified, but they might conceivably have been factually inaccurate, in which case an objection could have been raised.

The argument made by Joseph's brothers is much more open to debate. The steward might have argued that they returned the money they found out of some motive(s) other than the sheer compulsion of their honest natures: (a) to liberate their brother Simeon, which had been kept hostage (see Genesis, 42:24 and 43:23); or (b) because the famine in Canaan forced them to come back to Egypt (see 43:1); or even (c) because they feared eventual pursuit and retaliation; or simply (d) because the silver cup, being a tool for divining purposes, had more value than the sacks full of money, and thus tempted them to take more risks.

We accept the brothers' argument, because we believe that their honesty proceeded from their exceptional fear of God (irrespective of any more down to earth concerns), but it is not unassailable. Clearly, the empirical foundations of the major premise are rather complex, and an additional complication is the rather abstract psycho-ethical concept (namely, honesty) it involves. With regard to the minor premise, about the restitution of money - that was a straightforward observation of a singular physical event. In any case, this example well illustrates the inductive issues which may underlie an a-fortiori argument.

In the case of the argument by Moses concerning the stiff-neck and rebellion of the children of Israel, the major premise might be construed as a generalization from common experience. We know that children are less well behaved in the presence of their parents or school-teachers than in their absence, and similarly that people follow their leaders more strictly when their leaders' backs are not turned - and on this basis, the postulated major premise seems reasonable. But it might well be argued that though this is more often than not true, it is not always true (the children of Israel are indeed requested by Moses to make it untrue!) - and thus put the whole argument in doubt, or at least make it probable rather than necessary. As for the minor premise, it could be viewed as an overly severe evaluation of the behavior of the children of Israel - there is a subjective aspect to it.

We need not belabor the matter further. All this goes to prove, *not* as some logicians have claimed that a-fortiori argument is in principle without formal validity, but that it is often difficult to find solid material grounds for its effective exercise. It is thus understandable why Rabbinical legislators have usually regarded *qal vachomer* arguments as insufficient in themselves to justify a law, *unless supported by the authority of tradition.*

6.   Rabbinic Formulations

An important test of our general forms of *qal vachomer*, is their applicability to *the formulation of a-fortiori argument traditionally made in the Rabbinic literature.* Some logicians, like R. Luzatto (also known as the Ramchal), have a pretty large concept of *qal vachomer*, which includes any kind of scale of comparison as the effective middle term[5]. However, most authors seem to limit their concept to one specific kind of middle term,

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5 We might also mention a description proposed by Maccoby, "if something is known about one thing which has a certain quality in relatively 'light' form, then it must be true 'all the more so' of some other thing that has the same quality in a relatively 'heavy' form". This description is incomplete in various ways, but at least does not limit itself to legal issues.

namely the concept of 'legal restriction'. Thus, for instance, R. Chavel (P. 27, n. 106.) describes the argument as follows:

> *A form of reasoning by which a certain stricture applying to a minor matter is established as applying all the more to a major matter. Conversely, if a certain leniency applies to a major matter, it must apply all the more to the minor matter.*

R. Feigenbaum's description (p. 88) is even clearer, as the following quotation shows. (Note that we are effectively dealing with a scale of modality, and with nesting of modalities within modalities.)

> a) *Any stringent ruling with regard to the lenient issue must be true of the stringent issue as well;*
>
> b) *any lenient ruling regarding the stringent issue must be true with regard to the lenient matter as well.*

The indefinition and apparent subjectivity of the concepts of 'lenient' and 'stringent' (or synonyms to the same effect) is important to note. They seem to refer to subjective/emotional reactions to laws; i.e. whether a law is felt by people as a further hardship or as a release from duty. If we suppose more formal definitions, and regard every law - positive or negative, i.e. an imperative or a prohibition - as "stringent", and every absence of law - i.e. ethical contingency, permission and exemption - as a "leniency", then we must be very careful in this context, as modal logic is involved, which has special syllogistic behaviour-patterns (notably, one cannot draw a conclusion from a first-figure major premise which is not positively or negatively necessary).[6]

Such special formulations are easily assimilated by our general theory of *qal vachomer* argument, as follows:

a) P generally implies more 'stringency for the practitioner' (=R) than Q implies,

nonetheless, Q is stringent (R) enough to imply 'the practitioner subject to a certain restriction (or not-subject to a certain liberty)' (=S),

all the more, P is stringent (R) enough for this same ruling to apply (S).

b) P generally implies more 'stringency for the practitioner' (=R) than Q implies,

nonetheless, P is *not* stringent (R) enough to imply 'the practitioner subject to a certain restriction (or not-subject to a certain liberty)' (=S),

all the more, Q is *not* stringent (R) enough for this same ruling to apply (S).

Note that both arguments are antecedental in form, and one is expressed positively and the other negatively. The extreme theses (P, Q) are legal rulings; their middle thesis (R) is the magnitude of burden, let us say, they impose on a practitioner, and their subsidiary thesis (S) is a third legal clause, itself evaluated as burdensome to a certain degree. If the smaller burden (Rq) includes the subsidiary (Rs), then so does the larger (Rp); and by contraposition, if the greater burden excludes the subsidiary, then so does the lesser. Note, for the sake of symmetry, that we could conceive of similar formulas in which the middle thesis (R) is 'leniency for the practitioner', provided the subsidiary thesis (S) likewise changes in polarity, becoming 'the practitioner is subject to a certain liberty (or not-subject to a certain restriction)'.

Such formulas may be objected to, firstly, on the ground of their limited concept: they are conceived specifically in relation to the severity or laxity of ethical propositions (legal rulings, in Rabbinical terminology), whereas a-fortiori is a much wider process, applicable to non-ethical propositions. Secondly, and more radically, these formulas involve a middle

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6   This matter requires further study, in relation to rabbinical formulations of a-fortiori argument concerning "leniency".

thesis ('burdensomeness', say) *too vague and diffuse* to enable a sure conclusion: the major premise must be *general*, and such generality can only be known by generalization or enumeration. If by generalization, the conclusion is at best probable; if by enumeration, we are begging the question (i.e. we had to know the desired conclusion beforehand).

For a law P may be burdensome in many respects and another law Q may be burdensome in many respects, and P may well be burdensome in numerically more respects than Q is burdensome; *even so*, the burdens of P may or may not include all the burdens of Q, and indeed the burdens of P and Q may not overlap at all! In other words, in principle (i.e. formally), the inference is not necessary without further specifications which somehow guarantee that the burdens of P include all those of Q. That is, the laws under discussion here, P and Q, have certain implicit material relations which must be brought out into the open.

Thus the above mentioned Rabbinical formulations of a-fortiori argument, are not only limited in scope (to ethical theses), but they cannot be considered as having formal validity (i.e. invariably guarantee inference). They are at best broad guidelines, which may occasionally be found inapplicable. Indeed, the Rabbis were aware of this problem, and did occasionally object to attempted such inferences by one of their colleagues, and claim that a stringency of Q did not necessarily apply to P or a leniency of P did not necessarily apply to Q. Effectively, they invalidated the major premise, denying it to be general and making it at best probable, by apposition of an acknowledged exception; and by this means, they inhibited application of *qal vachomer* reasoning to S, the new case under consideration.

7.   Conclusions

A final word, concerning a-fortiori argument in Talmudic and post-Talmudic rabbinic literature. The *language* actually used in such literature for a-fortiori reasoning is various, and according to *The Practical Talmud Dictionary* of four main types (as listed below). See also *Talmudic Terminology* (pp. 69-70), and other similar books on the subject.

a.   Various phrases with the word *din* (meaning logical judgment, usually a-fortiori), namely: *eino din she, din, dina* (Aram.)*, bedin, vedin hu, vehadin notein, vehalo din hu*.

b.   Variants of *kol sheken* (meaning 'all the more so'), namely: *kol sheken, kol deken* (Aram.)*, lo kol sheken*.

c.   The expression *al achat kamah vekamah* (meaning 'if in this case... how much more so in that other case'). This expression is reportedly used more in Hagadic than Halakhic contexts.

d.   And the defining expression *qal vachomer* (meaning 'leniency and strictness'; note that *qal* should more precisely have been *qol*, being a noun like *chomer*).

With regard to the *frequency of use* of this terminology, not having a concordance of post-Biblical literature, I cannot say with precision what it is in fact. If we refer to the Index Volume of the Soncino edition (1952) of the Babylonian Talmud, we find the entries enumerated below, which suggest a minimum of 137 arguments of the type concerning us. I say 'suggest', because the references are to page numbers, which may contain more than one argument of the same type; also, not having looked at them, I cannot guarantee that they are all legitimate cases. I would suspect offhand, on the basis of my minimal experience of Talmud study, that this list is incomplete (all the more so if we include the Commentaries).

A fortiori	52
A minori ad majus	31
Deduction, proofs by	2
Inference from minor to major	8
Kal wa-homer	34
Major, inference from minor to	8
Minor, inference from major to	2

In comparing Biblical and Talmudic/Rabbinic literature, certain trends are observable, with regard to the a-fortiori argument. First, with respect to *quantity*: the Tanakh records at least some thirty cases (which does not of course mean that there were not much more unrecorded cases); in the Talmud I would venture to guess offhand the number of cases to be in the hundreds, and if we look at later literature (for example, Rashi, who seems to have a predilection for the form), it appears very common there too.

Second, with respect to *quality*: the complexity and confidence of a-fortiori use is progressively greater; more complicated conditional arguments are used, more elements of the argument are left tacit. This has to do with the *level of theoretical support and linguistic sophistication*: the a-fortiori language of Biblical times is colloquial and general (undifferentiated if/then terminology is used, typical expressions like *ve-af-ki* occur in contexts other than a-fortiori); in Talmudic times, and thereafter, we find common use of expressions like *qal vachomer* or *kol sheken* which indicate a theoretical reflection (like the work of Hillel, Shammai, R. Akiba, or R. Ishmael), and constitute a much more specialized lexicon.

I would like to point out that the *absence* in the whole Bible of such technical expressions would tend to belie the anachronistic thesis that Talmudic-style *pilpul* (more or less logical argumentation for interpretative purposes) existed in an already highly developed form in Biblical times. Had, say, king David already had a similar intellectual context, and studied daily in a similar manner (as some commentators later claimed), would he not have tended to use an equally explicit vocabulary, even in his everyday discourse (as is the case with Rabbis, scholars and students even today)?

That is, the claim that the gift of the Torah at Sinai included a ready-made oral equivalent of the Talmud and later writings, with all the accessory hermeneutic principles more or less clearly implied, does not seem confirmed by the aforegoing observations. *Absence of evidence is of course not proof to the contrary, but it weakens a thesis somewhat.* The alternative theory, that consciousness or at least verbalizing of logic underwent a historical *development* after Sinai seems, in the light of the above, more credible. On the other hand, the above observations tend to confirm the tradition that all the books in the Biblical Canon are rather ancient.

In the last analysis, however, it is hard to say precisely when, between Biblical and Mishnaic times, the change in logical language occurred. The most likely hypothesis is that it occurred just where the extant written record places it: namely, more or less abruptly, in the way of a cultural revolution, during the formative century or two of the Mishnah (roughly, 1st century BCE to 1st century CE), continuing on through the centuries during which the Gemara was developed.

For, as is evident from its form and content, the intellectual reflection on logic, which gave rise to this language change and is manifest in it, did not occur in a vacuum, as pure philosophical theory, but as *ad hoc* response to the specific issues the Talmudic Rabbis encountered in formulating their legal thoughts and debates. This verbal reflection on

logic, like its legal context, must have been written down to some extent at about the same time as it was developed, for the simple reason that the human mind, even at its best, can only handle so much data by itself; after which it needs material supports.

Just as arithmetic calculation cannot develop far without pencil and paper, and eventually algebraic tools (and still further on, computers); and likewise endeavors like architecture are limited without geometrical drawing, and eventually theoretical equipment (and later still, more sophisticated technologies); so without the use of *written* words to solidify past stages of thought and debate, and eventually abstract reflection on the logical methodology underlying it, cogitation cannot credibly develop beyond a certain intellectual level.

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